Upute za rješavanje zadataka sa prvog parcijalnog, april 2012

GRUPA A

1. $I=-\frac{arc\,tg\,x}{1+x}\Big|_0^1+\int_0^1\frac{dx}{(1+x)\left(1+x^2\right)}=-\frac{\pi}{8}+\int_0^1\frac{dx}{(1+x)\left(1+x^2\right)};$

$$\frac{1}{(1+x)\left(1+x^2\right)}=\frac{\frac{1}{2}}{1+x}+\frac{1}{2}\cdot\frac{1-x}{1+x^2}$$

$$\int_0^1\frac{dx}{(1+x)\left(1+x^2\right)}=...=\frac{1}{4}\ln 2+\frac{\pi}{8}\Rightarrow I=\frac{\ln 2}{4}.$$

2. Jednačina tangente: $y=x$; Tražena zapremina je

   $V=V_1+V_2=\pi\int_0^2 x^2dx+\pi\int_2^4\left[x^2-(8x-16)\right]dx=...=\frac{8\pi}{3}+\frac{8\pi}{3}=\frac{16\pi}{3}.$

3. $F'_x=2x+\lambda(6x-2y)$, $F'_y=2y+\lambda(4y-2x)$. Ako oboje izjednačimo sa nulom imamo da je

   $-\lambda=\frac{2x}{6x-2y}=\frac{2y}{4y-2x}\Rightarrow...\Rightarrow x^2-y^2+xy=0\Rightarrow 2x^2-2y^2+2xy=0.$

   Sabirući jednačine $2x^2-2y^2+2xy=0$ i $3x^2-2xy+2y^2=5$ dobijemo $5x^2=5\Rightarrow x=\pm1$. Dalje slijedi: $x=1\Rightarrow 1-y^2+y=0\Rightarrow y_{1,2}=\frac{1\pm\sqrt{5}}{2}$; $x=-1\Rightarrow 1-y^2-y=0\Rightarrow y_{3,4}=\frac{-1\pm\sqrt{5}}{2}$;

4. Uvedemo polarne koordinate. Tada je $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\varphi\int_0^{\sqrt{\pi}}\rho\sin\rho^2d\rho=...=\pi.$

GRUPA B

1. $I=arc\,tg\frac{x}{2}\cdot\frac{\sqrt{\left(4+x^2\right)^3}}{3}\Bigg|_0^2-\int_0^2\frac{2}{3}\sqrt{4+x^2}dx=\frac{4\pi\sqrt{2}}{3}-\frac{2}{3}I_1;$

$$\int\sqrt{4+x^2}dx=\frac{x}{2}\sqrt{4+x^2}+2\ln\left(x+\sqrt{4+x^2}\right)\Rightarrow I_1=2\left[\sqrt{2}+\ln\left(1+\sqrt{2}\right)\right].$$

$$I=\frac{4}{3}\left[\sqrt{2}(\pi-1)-\ln\left(1+\sqrt{2}\right)\right].$$

2. Jednačina tangente: $y=3-\frac{1}{2}x$;

   $V=V_1+V_2;\ V_1=\pi\int_0^3\left[\left(3-\frac{x}{2}\right)^2-(9-3x)\right]dx=...=\frac{9\pi}{4};\ V_2=\pi\int_3^6\left(3-\frac{x}{2}\right)^2dx=\frac{9\pi}{4}.$

   $V=2\cdot\frac{9\pi}{4}=\frac{9\pi}{2}.$

3. $F_x' = 4x + 12y + \lambda(2x)$, $F_y' = 12x + 2y + \lambda(8y)$. Ako oboje izjednačimo sa nulom imamo

$$-\lambda = \frac{4x+12y}{2x} = \frac{12x+2y}{8y} \Rightarrow ... \Rightarrow -6x^2 + 24y^2 + 7xy = 0 / : x^2$$

$$\frac{y}{x} = t \Rightarrow 24t^2 + 7t - 6 = 0 \Rightarrow t_1 = \frac{3}{8}, t_2 = -\frac{2}{3}.$$

$$\frac{y}{x} = \frac{3}{8} \wedge x^2 + 4y^2 = 25 \Rightarrow ... \Rightarrow A\left(4, \frac{3}{2}\right),\ B\left(-4, -\frac{3}{2}\right).$$

$$\frac{y}{x} = -\frac{2}{3} \wedge x^2 + 4y^2 = 25 \Rightarrow ... \Rightarrow C(3,-2),\ D(-3,2).$$

4. Uvedemo polarne koordinate. Tada je $I = \int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int\limits_{0}^{\sqrt{\frac{\pi}{2}}} \rho \cos\rho^2 d\rho = ... = \frac{\pi}{2}.$